Heaps

Min Heap - a binary tree with two added conditions
1. the tree is complete
2. a node is less than both of its children

```
   1
  / \
 2   3
 / \ / \ / \
6 5 9 6 9 4 6 5 9 3
```

Is min heap? √ X X

operations
min: get the root
add: add node to maintain completeness
    then heapify up on node
heapify up: if a node is less than its parent then swap and heapify up on parent otherwise done

```
add
2
/ \ 3
5 4

up 2 3
/ \ 3
5 4

heapify up

add
2
/ \ 3
5 4

up 2 3
/ \ 3
5 2

heapify up

add
2
/ \ 3
5 4

up 2 3
/ \ 3
5 2

heapify up
```

6 7 9 6 7 9 2 6 7 9 4 6 7 9 4
remove min: replace the root with the rightmost leaf in the highest level, heapify down on the new root

heapify down: if the node's left child is less than the node and less than the node's right child then swap the node and its left child and heapify down on the left child
otherwise if the node's right child is less than the node and less than the node's left child then swap the node and its right child and heapify down on the right child
otherwise done

<table>
<thead>
<tr>
<th>remove</th>
<th>min</th>
<th>heapify down</th>
<th>heapify down</th>
</tr>
</thead>
<tbody>
<tr>
<td>swap</td>
<td>3</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>7 8 9</td>
<td>6 7 8</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>7 8</td>
<td>6 7 8</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>7 9</td>
<td>6 7 9</td>
</tr>
<tr>
<td>Operation</td>
<td>BST</td>
<td>AVL</td>
<td>Min Heap</td>
</tr>
<tr>
<td>---------------</td>
<td>-----</td>
<td>-----</td>
<td>----------</td>
</tr>
<tr>
<td><strong>add</strong></td>
<td>$O(n)$</td>
<td>$O(\log n)$</td>
<td>$O(\log n)$</td>
</tr>
<tr>
<td><strong>find</strong></td>
<td>$O(n)$</td>
<td>$O(\log n)$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td><strong>find min</strong></td>
<td>$O(n)$</td>
<td>$O(\log n)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td><strong>remove min</strong></td>
<td>$O(n)$</td>
<td>$O(\log n)$</td>
<td>$O(\log n)$</td>
</tr>
</tbody>
</table>
Priority Queue

add    remove head
linear  $O(n)$  $O(1)$
heap    $O(\log n)$  $O(\log n)$
array of queues  $O(1)$  $O(p)$ (p is the number of priorities)

heap sort
add all elements to a heap
then remove the min repeatedly

heap merge quick insertion bubble sort

$O(n \log n)$  $O(n \log n)$  $O(n^2)$  $O(n^2)$  $O(n^2)$

$\Omega(n \log n)$  $\Omega(n \log n)$  $\Omega(n \log n)$  $\Omega(n)$  $\Omega(n) \Omega(n^2)$