Spring 2006 CPSC270A: Algorithms
Home work 1

Assigned: Friday, January 20, 2006
Due: Monday, January 23, 2006

There are two problems on two pages.

**Problem 1** Consider the following algorithm:

```c
int gcd1 (int m, int n) {
    int answer;
    if (n == 0)
        answer = m;
    else
        answer = gcd (n, m % n); // % is the mod operator.
    return (answer);
}
```

Prove that the above algorithm correctly computes the gcd of any two positive integers, \( m \) and \( n \).
Problem 2 Consider the following algorithm:

```c
int gcd2 (int m, int n) {
    int answer;
    if (m == n)
        answer = m;
    else if (m < n)
        answer = gcd (n, m);
    else
        answer = gcd (n, m - n);
    return (answer);
}
```

1. Prove that the above algorithm correctly computes the gcd of any two positive integers, \( m \) and \( n \).

2. Find an expression, in terms of \( m \) and \( n \), for the number of distinct values, including \( m \) and \( n \), that will be passed as parameters to \( \text{gcd2} \) while computing the gcd of \( m \) and \( n \). Consider the following two examples discussed in class.
   (a) \( \text{gcd2}(4, 3) \).
```
gcd2(4, 3) \rightarrow gcd2(3, 1)
     \rightarrow gcd2(1, 2)
     \rightarrow gcd2(2, 1)
     \rightarrow gcd2(1, 1)
     = 1.
```

Thus the distinct numbers in the calls to \( \text{gcd2} \) are 1, 2, 3 and 4, and thus the total of these numbers is 4.

(b) \( \text{gcd2}(6, 2) \).
```
gcd2(6, 2) \rightarrow gcd2(2, 4)
     \rightarrow gcd2(4, 2)
     \rightarrow gcd2(2, 2)
     = 2.
```

Thus the distinct numbers in the calls to \( \text{gcd2} \) are 2, 4, and 6, and thus the total of these numbers is 3.