1. A vertex, \( v \), in an undirected, connected graph \( G \) is a cut-vertex iff the graph \( G - v \), i.e., the graph obtained by removing from \( G \) the vertex \( v \) and all edges incident on \( v \), is not connected.

The complement of a graph \( G \), denoted by \( \bar{G} \), is the graph that has the same vertex set as \( G \), and for any two vertices, \( x \) and \( y \), \( xy \) is an edge in \( \bar{G} \) iff \( xy \) is not an edge in \( G \).

Prove or disprove: If \( v \) is a cut-vertex of an undirected, connected, loopless graph \( G \), then \( v \) is not a cut-vertex of \( \bar{G} \).

2. Suppose \( v \) is a vertex of an undirected, connected, loopless graph \( G \). Prove or disprove: \( v \) has a neighbour in every component of \( G - v \).

3. Suppose \( G \) is a graph such that the degree of every vertex is at least \( k \). Prove or disprove: \( G \) contains a path of length at least \( k \). (Recall that a path in \( G \) of length \( t \) is a sequence of distinct vertices \( v_0, v_1, \ldots, v_t \) such that for each \( i, 0 \leq i < t \), \( v_i v_{i+1} \) is an edge in \( G \).)

4. Suppose \( G \) is a graph whose vertex set is the set of permutations of \( \{1, \ldots, n\} \), with two permutations \( a_1, \ldots, a_n \) and \( b_1, \ldots, b_n \) adjacent (connected by an edge) iff one permutation can be obtained from the other by interchanging a pair of adjacent entries. Prove or disprove: \( G \) is connected.