Root Lattices are Efficiently Generated

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Abstract

We show that root lattices are exactly those lattices whose relevant vectors are all equal in length. We define notions of "compatibility" between the euclidean metric and the shortest path metric in the infinite directed graph induced by a subset, G, of a lattice, i.e., the directed graph whose vertices are the points in the lattice, and whose arcs are the ordered pairs $(x, x+g)$, with x a lattice point and g a point in G. We present some ("easy to check for") criteria that a lattice and a subset of it must satisfy to ensure "compatibility" between the corresponding graphical and the euclidean metrics. We use these criteria to characterize, in more than one way, a set of "economically and efficiently generated" lattices, including root lattices. Our results include a "graph theoretic" characterization of root lattices as well. We also discuss, in brief, certain algorithmic considerations in the simulation of macroscopic physical phenomena in massively parallel computers based on suitable discretizations of euclidean space that led us to our graphical treatment of lattices.

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